Engineering Notes

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Scaling of Cushion Capacitance in Air Cushion Vehicle Model Testing

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Nomenclature

A_p = cross-sectional area of flexible bellows
C_c = cushion capacitance = $V_c/\gamma P_a$
Fr = Froude number
g = acceleration due to gravity
h = height of vehicle center of gravity above datum
h_c = height of cushion volume
L = reference length of vehicle, usually overall length
M = mass of craft
P_a = absolute atmospheric pressure
p_c = cushion gage pressure
R = whirling-arm radius
S_c = cushion support area
t = time
T_a = atmospheric temperature
U = forward speed of vehicle
V_c = volume of cushion
W = vehicle weight
γ = ratio of specific heats of air
θ_j = attitude angles of model
λ = scale ratio = L_m/L_f
ρ_a = atmospheric density
ϕ_m = angle subtended by model at center of whirling-arm facility

Subscripts

av = average
f = full-scale
m = model scale

Introduction

FOR some operating conditions the compressibility of the air in the cushion volume of an air cushion vehicle (ACV) plays a major role in the dynamics. The p_c generated by ACV's is usually less than 8 kPa; thus, air compressibility is accurately described by a pneumatic capacitance C_c, and for model testing, scaling C_c is an unresolved problem. Lavis et al.[1] noted that in a geometrically scaled model C_c is too small by a factor of λ. To date, three techniques for circumventing this difficulty have been proposed: 1) testing in a wind tunnel, where atmospheric pressure can be controlled,[1] 2) use of a whirling-arm facility to alter the effective acceleration due to gravity,[2] and 3) addition of pressure-dependent flexible volume.[3] The purpose of this Note is to use dimensional analysis to demonstrate that atmospheric temperature, not pressure, must be controlled, and to argue that the second method is impractical. It is suggested that, for a sufficiently large model, the third approach may be feasible.

Conditions for Scaling of Cushion Capacitance

The approach adopted here is a variation of the one described by Mantle.[4] Inspection of the differential equations describing ACV dynamics shows that such quantities as θ_j and h depend on the following variables:

$$\theta_j, h = f(t; L, W, g, \rho_a, C_c, U, C_i) \tag{1}$$

where the quantities C_i are additional parameters not required for the present analysis.

Following the usual pi-theorem argument, choose three parameters having independent dimensions as reference quantities. Mantle uses ρ_a, L, and U, but here because we wish to consider the case of zero U, we choose ρ_a, L, and W. Then the pitheorem guarantees that Eq. (1) can be replaced by

$$\theta_j, \bar{h} = \bar{f}(\bar{t}; \bar{g}, \bar{C}_c, C_i) \tag{2}$$

where the bar indicates a dimensionless equivalent. For the present discussion, the key groups are

Gravity scaling:

$$\bar{g} = \rho_a g L^3 / W \tag{3}$$

Capacitance scaling:

$$\bar{C}_c = C_c W / L^5 \tag{4}$$

Forward speed scaling:

$$\bar{U} = UL(\rho_a / W)^{1/2} \tag{5}$$

To obtain correct scaling in a model when both ρ_a and g are variable, the gravity scaling condition requires that

$$W_m / W_f = (\rho_{am} g_m / \rho_{af} g_f)\lambda^3 \tag{6}$$

The capacitance of the model cushion must then satisfy

$$C_{cm}/C_{cf} = (W_f/W_m)\lambda^5 = (\rho_{af} g_f / \rho_{am} g_m)\lambda^2 \tag{7}$$

if both W and C_c are to be simultaneously scaled. Equations (5) and (6), when combined, give the usual Froude scaling requirement for model velocity:

$$U_m / U_f = (\lambda g_m / g_f)^{1/2} \tag{8}$$

Finally, note that, since $W = p_c S_c$, the gravity scaling condition is equivalent to the cushion density parameter $\bar{p}_c = p_c / (\rho_a g L)$ proposed by Mantle.[4]

Implications for Model Testing

To explore the consequences of these rules, consider first the case in which $\rho_{am} = \rho_{af}$ and $g_m = g_f$. Equation (6) gives $W_m \propto L_m^3$ and, since $S_{cm} \propto L_m^2$, $p_{cm} \propto L_m$, as is well known.[4] According to Eq. (10), the capacitance scaling condition requires $C_{cm} \propto L_m^2$, but because in a geometrically scaled model $C_{cm} \propto V_{cm} \propto L_m^3$, a fundamental contradiction exists. This is the difficulty first pointed out by Lavis et al.[1] Its implications can

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be illustrated by the following argument. In a model the region corresponding to the payload-carrying part of the full-scale vehicle does not usually serve any specific purpose; thus, it can be used as additional volume for capacitance. Since $V_c = h_c S_c$, and $S_c \propto L^2$, then capacitance scaling amounts to the requirement that $h_{cm} = h_{cf}$.

The conclusion that the capacitance scaling problem cannot be resolved by altering the atmospheric pressure alone follows immediately from Eq. (7) and the definition of C_c:

$$\frac{V_{cm}}{V_{cf}} = \frac{P_{am}\rho_{af}}{P_{af}\rho_{am}} \cdot \frac{g_f}{g_m} \lambda^2 = \frac{T_{am}}{T_{af}} \frac{g_f}{g_m} \lambda^2 \tag{9}$$

Since $P_a/\rho_a = RT_a$, Eq. (9) indicates that altering T_a is crucial for capacitance scaling.

Equation (9) suggests that a cryogenic wind tunnel might work. However, with $g_m = g_f$ and $V_{cm} \propto L_m^3$, $T_{am}/T_{af} = \lambda$. Cryogenic tunnels[6] use a minimum T_a of about 100 K, so that $\lambda \geq 0.33$. Given this constraint and the limited size of available facilities,[6] this approach is probably unrealizable.

Now consider ρ_a fixed, but g variable. Then, with $C_c \propto \lambda^3$ from Eq. (7),

$$C_{cm}/C_{cf} = \lambda^3 = (g_f L_m^2/g_m L_f^2)\lambda^2 \quad \text{or} \quad g_m/g_f = 1/\lambda \tag{10}$$

Then Eqs. (8) and (10) give

$$U_m/U_f = 1 \tag{11}$$

which is the scaling requirement obtained by Richardson[2] using an argument based on the aeroelasticity triangle of Collar.[5]

Using a whirling-arm facility to alter g has a severe limitation. It was pointed out by Richardson[2] that, if the effective gravity is provided by rotating the model on an arm of R, then $g_m = U_m^2/R$ and

$$Fr \triangleq U_m/(L_m g_m)^{1/2} = (R/L_m)^{1/2} = \phi_m^{-1/2} \tag{12}$$

This places a severe lower limit on test speed. Richardson[2] suggests that $\phi = 30$ deg is practical maximum, which requires $Fr_m \geq 1.38$ as a lower limit. For a vehicle such as an amphibious cargo transporter (LACV30), $L_f = 23.3$ m; thus, this imposes a minimum equivalent $U_f = 17.8$ m/s; such vehicles have normal maximum service speed of about 18 m/s (35 kn). Hence, one can rule out the use of a whirling arm for all but a very restricted range of conditions.

Scaling Capacitance with a Flexible Volume

Mathematical models of cushion dynamics always include a flow conservation equation of the form

$$C_c \dot{p}_c + \dot{V}_c = Q_c - Q_a \tag{13}$$

where Q_c and Q_a are the volume fluxes into and out of the cushion, respectively. The term $\dot{V}_c$ is associated with both vehicle motion and flexible skirt deformation under the action of p_c. With $V_c = V_c(p_c, h, \theta_j)$, Eq. (13) can be rearranged to

$$(C_c + C_{cp})\dot{P}_c + \dot{V}_{cg} = Q_c - Q_a \tag{14}$$

where $C_{cp} = \partial V_c/\partial p_c$ is an extensibility capacitance, and $\dot{V}_{cg}$ is the volume variation caused by the geometric dependence of V_c on h and θ_j. The total capacitance can thus be adjusted by incorporating a flexible volume to control C_{cp}.

A suitable flexible volume might be obtained by using a bellows system in which the extension is controlled by mechanical springs. Examples illustrating the use of this technique to scale capacitance to a test vehicle used on a 43.6-m-diam circular track at the University of Toronto are now presented for this LACV30 and a 270-ton ice-breaking platform (ACIB). For the test vehicle the minimum value of M_m is about 900 kg. Hence, for the LACV30 with planform dimensions of 23.3 × 11.2 m and fully loaded at 52 tons, the gravity scaling condition gives $\lambda = 0.259$ and model planform dimensions of 6.02 × 2.90 m. This is significantly larger than the current dimensions of 4.08 × 2.03 m but is nevertheless feasible. For the LACV30, $C_c = 2.1 \times 10^{-3}$ m³/Pa approximately, which, according to Eq. (7), scales to 1.4×10^{-4} m³/Pa for $\lambda = 0.259$. The capacitance of the scaled cushion volume is about 3.8×10^{-5} m³/Pa. The extra capacitance can be supplied by a bellows having $A_p = 2.4$ m² and a total spring stiffness of 550 N/cm. Assuming that this stiffness is divided among a large number of springs and that the moving element is made from a lightweight composite material sandwich panel, a total moving mass = 2 kg is feasible. Thus, the natural frequency of such a system is about 26 Hz. To avoid phase distortion, it is desirable to limit maximum model test frequencies to about one-half of this. A 13-Hz frequency limit implies that, when the LACV30 model is tested at $U_m = 9.6$ m/s, corresponding to $U_f = 19$ m/s (37 kn), disturbance wavelengths should be no shorter than 12% of craft length.

Scaling the 24.7 × 24.7 m ACIB operating at its minimum $M_f = 175$ tons to $M_m = 900$ kg requires planform dimensions of 4.26 × 4.26 m, or $\lambda = 0.172$. The C_{cf} is about 5.9×10^{-3} m³/Pa, which implies that C_{cm} must be 1.8×10^{-4} m³/Pa. The model volume gives $C_c = 3.27 \times 10^{-5}$ m³/Pa; thus, the extra to be supplied by the bellows requires $A_p = 0.8$ m² and a total spring stiffness of 45 N/cm. For a panel mass of 0.75 kg, the natural frequency of the system is 12.3 Hz. Assuming a maximum $U_f = 5.2$ m/s (10 kn), which scales to $U_m = 2.1$ m/s, and maximum desirable full-scale frequencies of 6 Hz, disturbances having wavelengths down to 8% of model length could be investigated.

A key requirement is that the device must have enough variable volume to accommodate the pressure fluctuations generated by vehicle motion. Simulations were used to obtain an estimate of the amount required.[3] For the bellows system, the total panel movement Δw required is

$$\Delta w = \Delta V_p/A_p = A_p^{-1} \int (\partial V_c/\partial p_c)\dot{p}_c \, dt \approx A_p^{-1}(\partial V_c/\partial p_c)_{av}\Delta p_c \tag{15}$$

The integral is taken over that part of a cycle for which $\dot{p}_c$ does not change sign. Assuming a pitch-heave response to an initial condition disturbance, this analysis predicts that the maximum Δp_c is about 0.6 kPa for a vehicle having $M = 900$ kg and a 4 × 2 m planform. If the LACV30 model experiences a similar fluctuation, then, with $A_p = 1.2$ m², the total Δw required is 5.0 cm, and for $A_p = 2.4$ m², $\Delta w = 2.5$ cm. A more severe test would be a heave oscillation involving skirt-ground contact leading to flow shutoff. Then Δp_c may peak at values near $1.5\, p_{ce}$, in which case for $A_p = 1.2$ m², $\Delta w = 10$ cm and for $A_p = 2.4$ m², $\Delta w = 5$ cm. These are probably feasible. For the ACIB the Δw required for the two cases described above are 11 and 24 cm, respectively; if A_p is doubled to 1.6 m², then $\Delta w = 5$ and 12 cm. Here the larger area reduces the required travel to the levels that might be practical.

Conclusion

Of the three proposals for scaling the pneumatic capacitance of an air cushion discussed here, it appears that the only realistic prospect is to use a bellows system or equivalent flexibility capacitance on a relatively large scale model. Use of large-scale models also greatly aids the task of scaling the flexible skirts.

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Lift-Curve Slope for Finite-Aspect-Ratio Wings

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Introduction

RECENTLY derived exact solutions for the lift produced by an elliptic planform flat plate in steady incompressible potential flow are applied to improve the commonly used approximations for the lift curve slope of finite aspect-ratio wings. An excellent approximation is derived for the initial lift-curve slope when the aspect ratio ≤ 2. For large aspect ratios, the lift-curve-slope equation is improved so that it can more accurately calculate the corresponding two-dimensional lift-curve slope from the wind tunnel test data of a finite-aspect-ratio wing.

Applications

The usual approximation for the lift curve slope C_{L_α} for incompressible flow on large aspect ratio wings is written as

$$C_{L_\alpha} = a_o/1 + \frac{57.3a_o}{\pi A e_1} \tag{1}$$

where the aspect ratio $A = b^2/S = (\text{span})^2/(\text{ref. area})$, $a_o \approx 0.1/$deg is the experimental lift curve slope for two-dimensional flow, and $e_1 \approx (1+\tau)^{-1}$ is either the experimental or theoretical correction for non-elliptical wing loadings. It will now be shown that a_o should not be included in the denominator so that a simpler and more accurate representation is given by

$$C_{L_\alpha} = \frac{a_o}{1 + (2/A)(1+\tau)} \tag{2}$$

This corresponds to the relations first given by Prandtl[1] with $\tau = 0$ for the first approximation to the elliptical spanwise lift distributions (also see Refs. 2 and 3).

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The correct Eq. (2) is based on idealized lifting-line theory that has the wing downwash angle α_i remain constant along the entire wing span so one can write

$$C_L = 2\pi(\Delta\alpha - \alpha_i) \approx 2\pi[\Delta\alpha - (C_L/\pi A)(1+\tau)] \tag{3}$$

where $\Delta\alpha = (\alpha - \alpha_{L=0})$ is the angle of attack measured from zero lift, and τ is the correction to lifting-line theory for a non-elliptical spanwise lift distribution. Actually, α_i is constant only for the ideal elliptical load distribution when $\tau = 0$, so the τ correction is only an approximation based on 2π, which is not affected by a_o, a viscous real fluid effect on the two-dimensional flow. This approximation is clearly evident when one notes that if $\tau \neq 0$, then the induced drag coefficient becomes

$$C_{D_i} = (C_L^2/\pi A)(1+\delta) < C_L\alpha_i = (C_L^2/\pi A)(1+\tau) \tag{4}$$

For example, if $A = 6$ for a rectangular planform wing, then Glauert[3] calculated that $\delta = 0.046$ and $\tau = 0.163$ so that $C_{D_i} = 0.0555\ C_L^2 < C_L\alpha_i = 0.0617C_L^2$. This effect of $\tau \neq 0$ was first pointed out by Karman and Burgers.[4] The use of $a_o = 0.1$ in the denominator of Eq. (1) erroneously reduces the aspect ratio correction and increases C_{L_α}. For the same example of a rectangular wing with $A = 6$, the better approximation from Eq. (2) is $C_{L_\alpha} = 0.072$ for $a_o = 0.1$, whereas Eq. (1) gives a higher value of 0.074. The replacement of 2π by a_o is only justified in the numerator of Eq. (2) because it represents an additional correction for an increasing boundary layer thickness followed by flow separation, usually on the wing's upper surface, and is not related to $\alpha_i = (C_L/\pi A)(1+\tau)$.

It is important to note that the numerical values of τ are based on lifting-line theory, and there is an additional correction, even for the ideal elliptic planform wing, when one introduces the more exact lifting-surface theory. For example, Kida and Miyai[5] have shown that for the ideal potential flow about a thin flat plate with an elliptic planform, the second approximation for $A \gg 4/\pi$ can be written as

$$\tau = (8/\pi^2 A)[\ln(\pi A) - 1] + 0(1/A^2) \tag{5}$$

The best comparison for the restrictions upon Eq. (5) are given by Hauptman and Miloh[6] who derived the first explicit relations for the elliptic flat plate as given by

$$C_{L_\alpha} = 4/\left[K + \frac{E^2(h)}{k + (\arcsin h)/h}\right]; \quad \left(k = \frac{4}{\pi A} \leq 1\right) \tag{6}$$

$$C_{L_\alpha} = \left(\frac{32}{8+\pi^2}\right) = 1.791; \quad \left(A = \frac{4}{\pi}; k = 1; h = 0\right) \tag{7}$$

$$C_{L_\alpha} = \frac{4k}{1 + E^2(h)/[1 + (k^2/h)\ln(1/k + h/k)\ ]}; \quad (k = \pi A/4 \leq 1) \tag{8}$$

For Eq. (6), $k < 1$, $A = (4/\pi k)$ and $h^2 = (1 - k^2)$. For example, if $k = (\text{mid-chord/span}) = 1/2$ then $A = (8/\pi)$, $h = (\sqrt{3}/2)$ and $E(h) = E(60\ \text{deg}) = 1.211$, where E is the complete elliptic integral of the second kind. These equations are given here for completeness because Eq. (6) is incorrectly printed on page 771 (Eq. 34) of Ref. 7, and there is a misprint of Eq. (8) on pages 54 and 55 (Eq. 65a) of Ref. 6, where the term ahead of the $\ln$ term should be (k^2/h) and not (k/h).

The calculations from these equations are compared in Table 1, which also includes a very remarkable approximation derived by Helmbold,[8] which he gave as

$$C_{L_\alpha} = \frac{2\pi A}{2 + (4 + A^2)^{1/2}} \tag{9}$$